Twisted Square

Student Objectives

Students answer their driving question and investigation questions through collaborative research, and then deliver their results to their classmates in a concise. The student will:

Mandatory

- ♦ Construct a series of larger squares with smaller, twisted squares inscribed in them according to a sequence of instructions.
- ♦ Experiment with geometric transformations of triangles, squares, and other polygons in the plane.
- ♦ Describe the rotations and reflections that carry a triangle or square onto itself.
- ♦ Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using graph paper, tracing paper, or geometry software.
- ♦ Determine the area of composite two-dimensional figures comprised of a combination of triangles, trapezoids, and other regular polygons.
- ♦ Prove the congruence of two or more polygons visually.
- ♦ Use trigonometric ratios and the Pythagorean equation to solve right triangles.
- ♦ Make connections between the mathematical concepts in this project and real life applications.
- ♦ Present results and conclusions to the class.

TEKS Objectives

This project correlates to the following TEKS objectives:

(c)1 The student uses mathematical processes to acquire and demonstrate mathematical understanding. The student is expected to:

B. use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and reasonableness of the solution;

C. select tools, including real objects, manipulatives, paper and pencil, and technology as appropriate, and techniques, including mental math, estimation, and number sense as appropriate, to solve problems;

D. communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate;

(c)11 Two-dimensional and three-dimensional figures. The student uses the process skills in the application of formulas to determine measures of two- and three-dimensional figures. The student is expected to:

B. determine the area of composite two-dimensional figures comprised of a combination of triangles, parallelograms, trapezoids, kites, regular polygons, or sectors of circles to solve problems using appropriate units of measure;

C. apply the formulas for the total and lateral surface area of three-dimensional figures, including prisms, pyramids, cones, cylinders, spheres, and composite figures, to solve problems using appropriate units of measure; and

D. apply the formulas for the volume of three-dimensional figures, including prisms, pyramids, cones, cylinders, spheres, and composite figures, to solve problems using appropriate units of measure.

This project correlates to the following Common Core State Standards:

HSG.GMD.A Explain volume formulas and use them to solve problems:

1 Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone.

3 Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.

HSG.GMD.B Visualize relationships between two-dimensional and three-dimensional objects:

4 Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.

HSG.MG.A Apply geometric concepts in modeling situations:

1 Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).

2 Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).

3 Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).

HSG.SRT.C Define trigonometric ratios and solve problems involving right triangles:

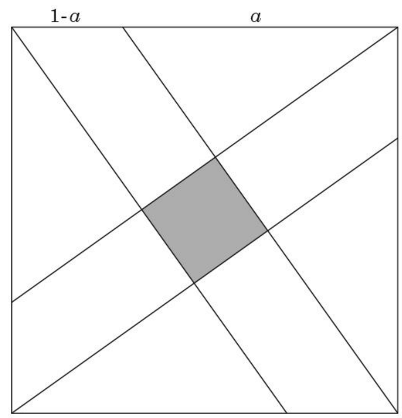
6 Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of

trigonometric ratios for acute angles.

8 Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.

Project Background

Given a unit square, a second—smaller—“twisted” square can be created by connecting each of the four vertices of the unit square to a point on the opposite (nonadjacent) side. The initial choice of a vertex and corresponding point on the nonadjacent side is arbitrary; for example it can be made to divide the opposite side into two segments that are one-third and two-thirds of a unit respectively, or one-fourth and three-fourths of a unit respectively, or any proportional length a to 1 - a. Once the point has been chosen on one side of the unit square, congruent lines should be constructed from each of the remaining vertices The result is a smaller square that is rotated some number of degrees off the axis of the original square.



The original square is similar to the smaller, rotated square. By definition, all squares are similar to each other because their corresponding angles are congruent—that is, all angles are 90º, and their corresponding side lengths are proportional. A less formal expression of this concept is that all squares are similar to each other because they have the same shape but are not necessarily the same size.

In this project students will first draw and cut out triangles and these use these to form three squares of 35 units in length, 25 units in length, and 20 units in length, respectively. They will use these constructions to review and apply the skills necessary before introducing a more complex problem. Students next construct a square that is 10 units on a side, and then connect the midpoint of each side to the vertices to construct the smaller, rotated square within it. They will use their construction to investigate the properties of congruence and similarity, and will experiment with transformations that enable them to demonstrate congruence visually by superimposing a polygon on one it is identical to.

Students will apply the Pythagorean equation, the sine and cosine relationships for right triangles, and the Angle-Angle Theorem for proving similarity of triangles to solve for missing measures of a variety of polygons. They will apply prior knowledge of algebra to prove lines either parallel or perpendicular based on their slopes being the same or having a product of −1. Students will be asked in the Extension and Real Life Applications questions to solve a quadratic equation and choose the most logical solution to determine the point that makes the ratio of the sides of the larger square such that the inner square’s area will be half that of the larger square.

Materials and Special Preparation

Mandatory Equipment

- Graph paper, several sheets
- Construction paper, several sheets
- Colored pencils
- Ruler or straightedge
- Scissors
- Tape

Optional Equipment

- Dynamic geometry software
- Drawing compass

The student handout for this activity lists the mandatory and optional equipment shown above that can be used as part of the student's project. Please review these items prior to providing them to the student.

Safety and Maintenance

Be certain that students add this important safety precaution to their normal classroom procedures:

- Handle all sharp objects carefully, including drawing compasses, scissors or craft/hobby knives.

Time Guidelines

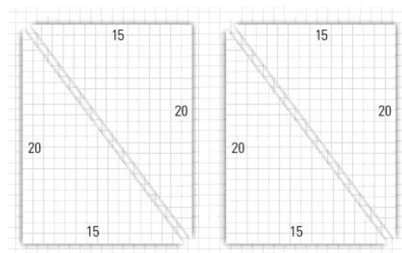
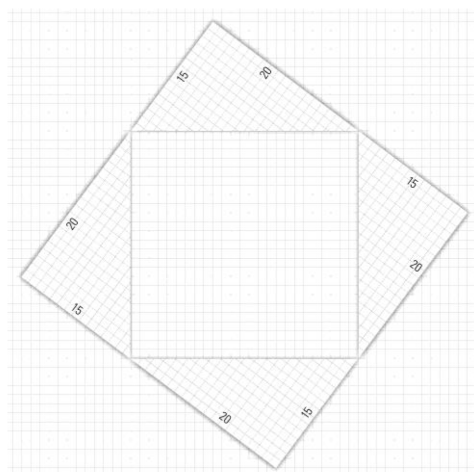
Students working together in a group should be able to complete Investigation Questions 1 – 16 (Easy to Moderate degree of difficulty) within one 50-minute class period. A second class period will be necessary to complete the remaining Investigation Questions.

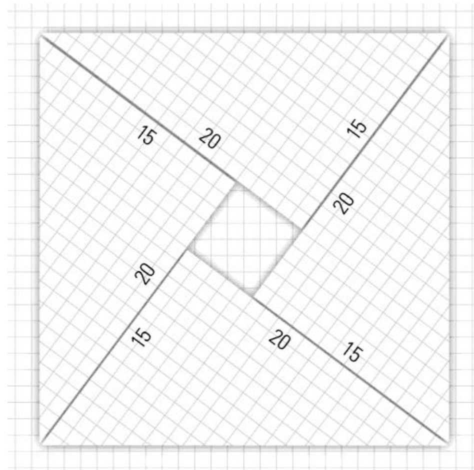
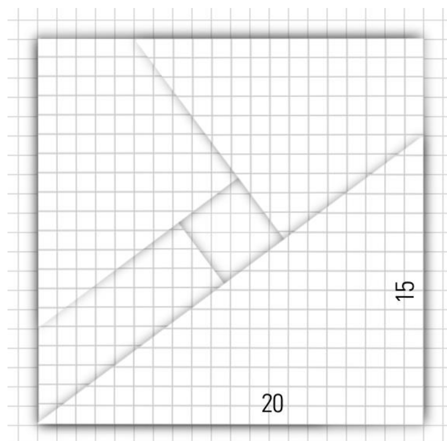
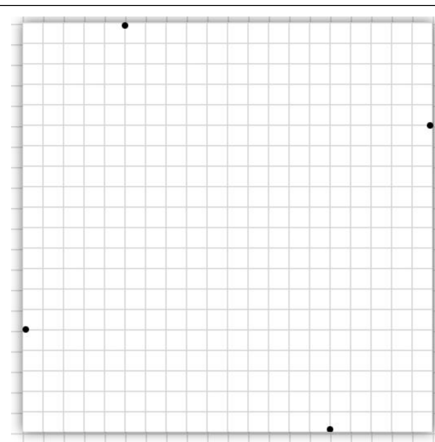
Presentation Guidelines

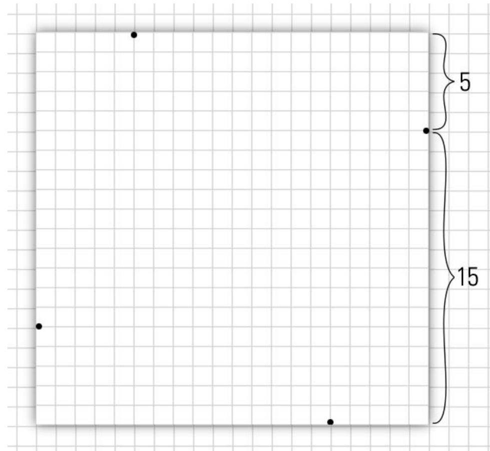
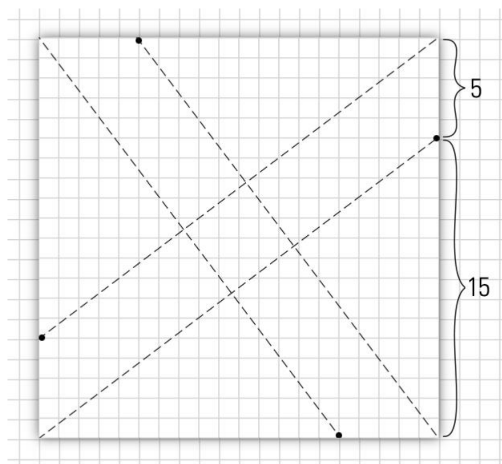
Use the following guidelines to assess the student's preparation, content knowledge, and presentation delivery.

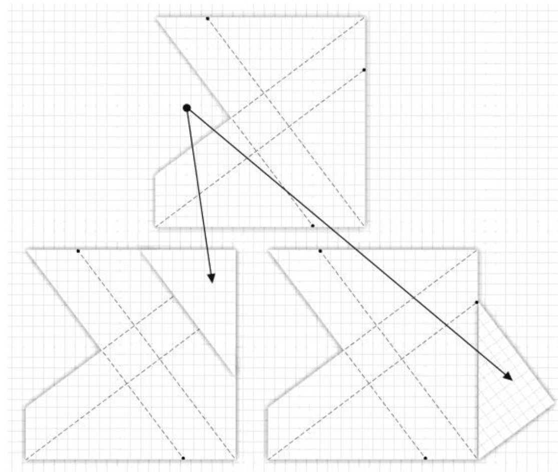
Investigation Guidelines

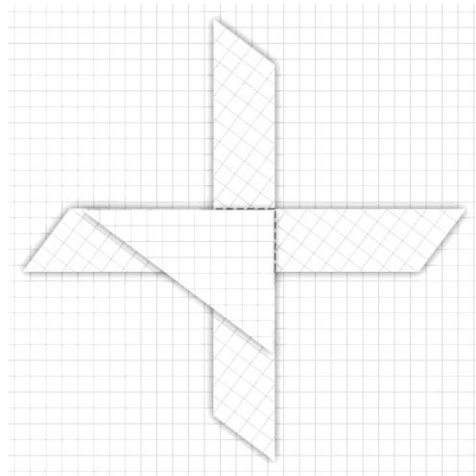
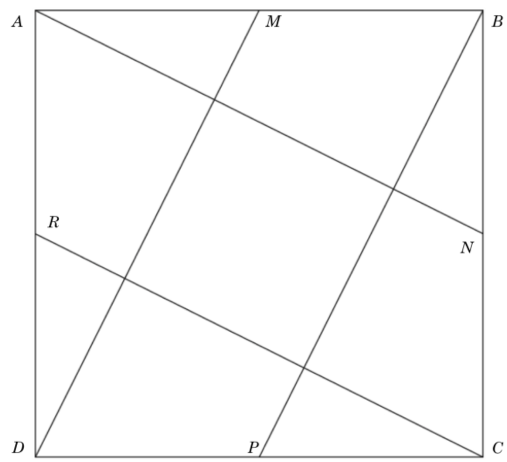
Follow these guidelines to assess each of the investigation questions from three parts of the student project handout. The third column indicates the difficulty level of the concept addressed in the investigation question.

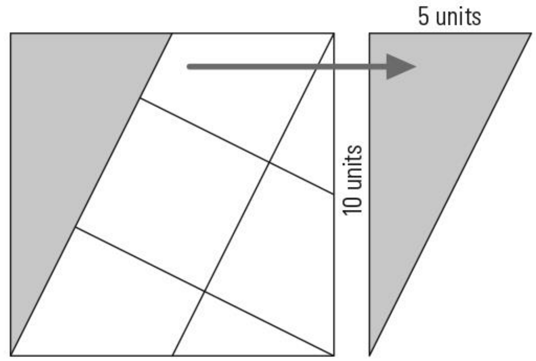
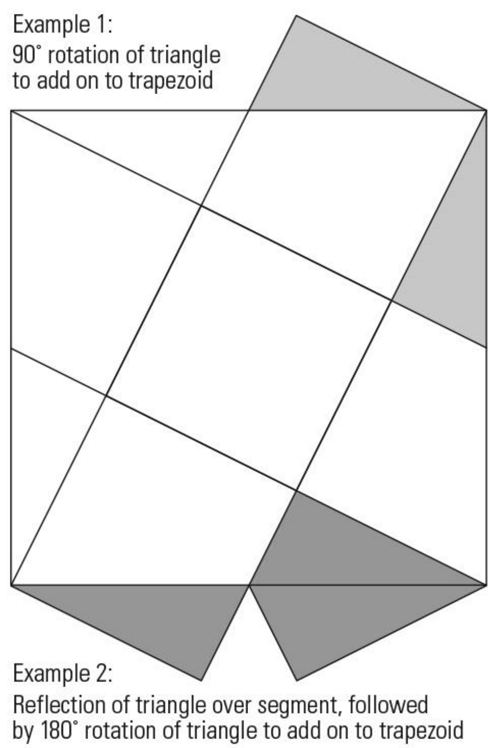
Investigation Questions	*Suitable Response*	*Difficulty*
1. Use graph paper to draw and cut out four right triangles whose perpendicular sides are 15 units and 20 units respectively.		Easy
2. Arrange the four right triangles to form a square with sides each 35 units in length. Without measuring, how can you determine the length of one side of the inner square? What is its length?	 Find the length of inner square's side using the Pythagorean equation: $c^2 = a^2 + b^2$ $c^2 = 15^2 + 20^2$ $c^2 = 625$ $c = \sqrt{625}$ $c = 25$	Easy

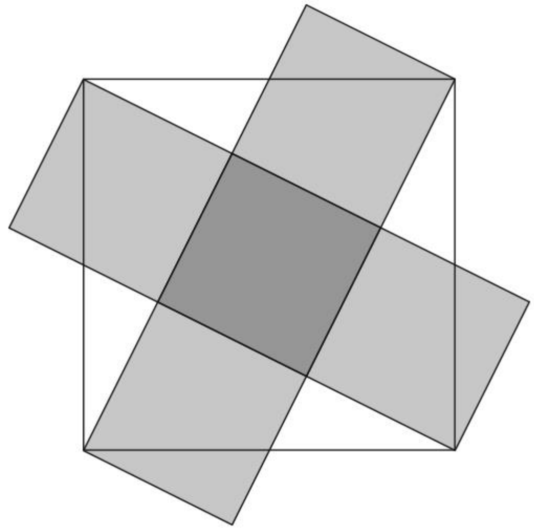
Investigation Questions	***Suitable Response***	***Difficulty***
3. In the previous step you used four right triangles to form the largest square possible. Rearrange the four right triangles to form a smaller square whose sides are less than 35 units in length, without overlapping the triangles. What is the length of a side of the twisted inner square? How can you determine this without measuring the side?	 The length of the square's side is 5 units. This determined by subtracting the shorter side of the right triangle from the longer side: 20 units – 15 units = 5 units.	Moderate
4. Form an even smaller twisted square by rearranging the triangles again. This time overlap the four right triangles to form a square whose sides are 20 units in length. Can the length of the inner square's side be found using the same method as for the two previous inner squares? Justify your answer.	 The same method cannot be used to find the length of the square's side because addition or subtraction of sides of the larger right triangles does not yield the length of a segment congruent to the square's side.	Moderate
5. Draw and cut out a square that is 20 units on each side.		Easy

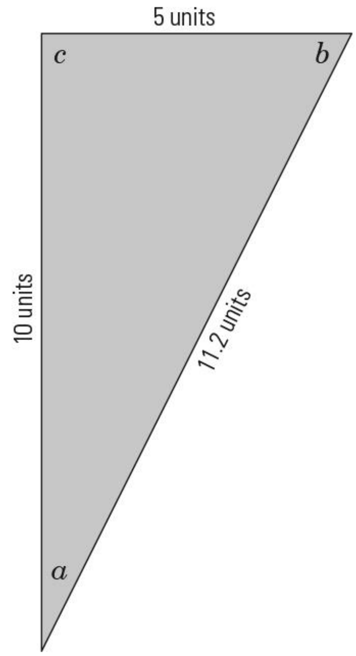
Investigation Questions	Suitable Response	Difficulty
6. Use one of the right triangles to divide each side of the 20 x 20 square into two segments – one segment of 15 units, one segment of 5 units. Mark a point at each side to indicate this division.		Moderate
7. Connect each vertex of the square to the point on the opposite side by drawing a segment along the hypotenuse of the right triangle.		Moderate

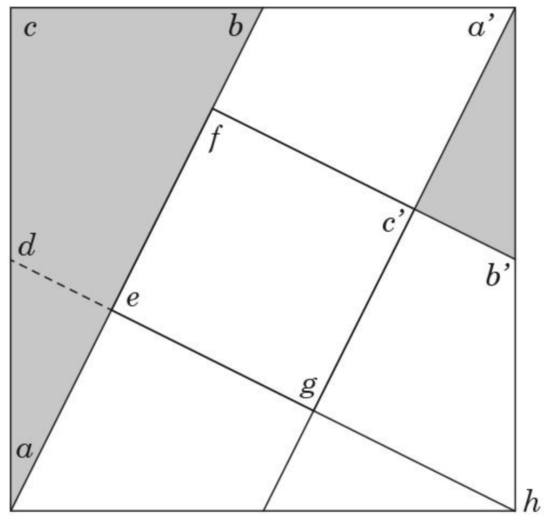
Investigation Questions	Suitable Response	Difficulty
8. Cut out one or more of the small triangles. Later in this activity you will derive the area of the "twisted square" formed in a construction similar to this. Part of the derivation requires you to apply the following skills: calculating acute angles in triangles, and solving for unknown side lengths in right triangles. Practice these skills with this construction. a. Using colored pencils identify the opposite and adjacent sides of one of the acute angles in the right triangles. b. To find the acute angles of the right triangle, which of the following trigonometric relationships do you need to apply: sine, cosine, or tangent? c. Calculate the acute angles and show your steps.	 b. The trigonometric relationship that relates the opposite and adjacent sides of an acute angle is the tangent. $\tan\theta = \frac{opp}{adj}$ c. $\tan\theta = \frac{15}{20}$ $\tan\theta = 0.75$ $\tan^{-1}\theta = 36.9° \approx 37°$ d. Use the sum of the interior angles of a triangle: $m\angle 1 + m\angle 2 + m\angle 3 = 180°$ $180° - m\angle 1 - m\angle 2 = m\angle 3$ $m\angle 3 = 180° - 90° - 37°$ $m\angle = 53°$	Moderate

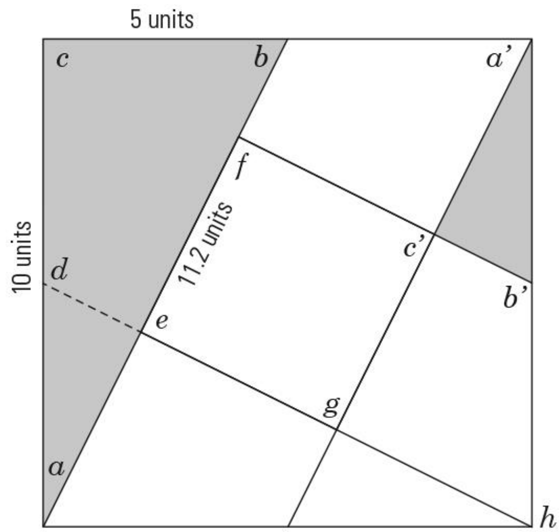
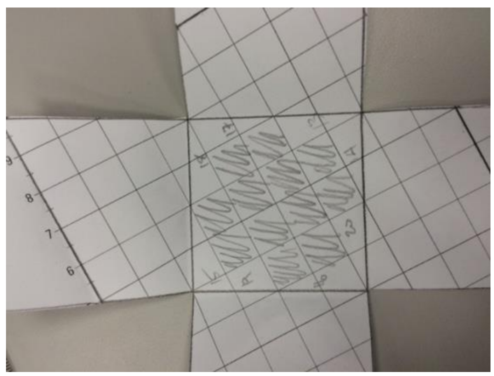
Investigation Questions	*Suitable Response*	*Difficulty*
9. Use the right triangles to prove visually that the inner twisted square is actually a square.		Easy
10. Define a square ABCD, with side length of 10 units. Name the midpoints M, N, P, R, of sides AB, BC, CD, and DA, respectively. Draw the segment that joins vertex A to midpoint N. Do this for the remaining vertices and midpoints. Use the grid provided.		Easy
11. What other types of polygons have been formed inside the 10 unit² square as a result of constructing the smaller square?	Right scalene triangles and trapezoids have been formed in addition to the smaller square.	Easy

Investigation Questions	*Suitable Response*	*Difficulty*
12. What is the length, in units, of the segment that connects the larger square's vertex to the midpoint on the opposite side? Show your method and calculations for determining the length.	 Use the Pythagorean equation for right triangles, with sides 10 and 5 units, to solve for the hypotenuse: $a^2 + b^2 = c^2$ $c = \sqrt{a^2 + b^2}$ $c = \sqrt{10^2 + 5^2}$ $c = \sqrt{125}$ $c = 11.180$ $c \approx 11.2$	Moderate
13. Describe at least one way in which one of these polygons can be transformed to create a new square congruent to the "twisted" square.		Moderate

Investigation Questions	*Suitable Response*	*Difficulty*
14. Use transformations to construct additional squares congruent to the smaller square, to form a 5-square cross that is similar to a "plus sign" or the symbol on the flag of Switzerland.		Moderate
15. Using additional graph paper onto which you trace the original square ABCD and any transformations, prove visually that all of the triangles created by transformations are congruent both to one another and to the small triangles within the large square ABCD.	Cut out pairs of triangles and lay one on top of another to demonstrate that each component of the top triangle has the same measure as (is congruent to) the corresponding component of the bottom triangle. This can also be done by folding the paper instead of cutting out the shapes.	Moderate
16. Using additional graph paper as needed, prove visually that all of the smaller squares are congruent.	Cut out the 5-square cross-shaped polygon. Crease the paper along each edge of the center square. Fold up the four "arms" of the cross so they lie flat on the center square. The squares are all the same size (corresponding parts are all congruent) because they cover each other without overlapping or having gaps.	Moderate

Investigation Questions	Suitable Response	Difficulty
17. The segment length calculated in Question 12 forms the hypotenuse of a right triangle. What is the number in degrees in the acute angles of this triangle? How can you determine the angle without measuring the angle directly?	 Use the trigonometric relationships sine and cosine, and the known lengths of the sides of the right triangle (from the previous question) to solve for the missing angles: $\sin a = \frac{opposite}{hypotenuse}$ $\sin a = \frac{5}{11.2}$ $\sin a = 0.446628$ $\sin^{-1} a = 26.5°$ $m\angle a = 26.5°$ $\sin a = \cos b$ $\sin a = 0.446429$ $\cos b = 0.446429$ $\cos^{-1} b = 63.5°$ $m\angle b = 63.5°$ Since angle *c* is a right angle (definition of a square), its measure is 90º. The sum of the internal angles of any triangle is 180º: $m\angle a + m\angle b + m\angle c = 180°$ $26.5° + 63.5° + 90° = 180°$ This serves as a further check on the method and calculations.	Difficult

Investigation Questions	Suitable Response	Difficulty
18. What theorem can you apply to prove that two triangles are similar? Apply this theorem to prove that any one of the larger triangles in square ABCD is similar to any of the smallest triangles you constructed within square ABCD.	 Use the Angle-Angle Theorem: If two pairs of corresponding angles in a pair of triangles are congruent, then the triangles are similar. We know this because if two angle pairs are the same, then the third pair must also be equal. When the three angle pairs are all equal, the three pairs of sides must also be in proportion. Prove that $\triangle abc \sim \triangle a'b'c' \sim \triangle ade$ **Proof** Given: The outer square and the inner square are defined to be squares. 1. $\angle dea \cong \angle b'c'a'$ by alternate exterior angles ($\overline{ba} \parallel \overline{a'g}$). 2. $\angle c'a'b' \cong \angle aed$ by alternate exterior angles ($\overline{ca'} \parallel \overline{a'h}$). 3. $\triangle ade \cong \triangle a'b'c'$ by the sum of the interior angles of a triangle is 180º. 4. $\triangle ade \sim \triangle a'b'c'$ by congruent triangles are also similar. 5. $\angle ead \cong \angle bac$ by reflexive property. 6. $m\angle aed = 90º$ by vertical angles with $\angle gef$ (given in definition of square). 7. $m\angle acb = 90º$ by definition of square. 8. $\angle ade \cong \angle a'b'c'$ by the sum of the interior angles of a triangle is 180º. $\triangle abc \sim \triangle a'b'c' \sim \triangle ade$ by Angle-Angle Theorem	Difficult

Investigation Questions	Suitable Response	Difficulty
19. Show two different ways to find the area of the smaller square. Verify your results by shading in the grid squares to sum the total area.	  Students may choose a variety of techniques to find the area of the small square; here are examples of two methods that are commonly used. *Method 1:* Use previous values for lengths of sides of Δabc and the fact that it is similar to $\Delta a'b'c'$ to solve for unknown sides of the smaller triangle, using proportions. Dimensions of the smaller triangle: Base = 2.232 units; Height = 4.464 units The height of the smaller triangle $\Delta a'b'c'$ is equal to the length of the square's side, so the area of the square is 4.464^2 units^2 or 19.93 units^2. *Method 2:* Use previous values for lengths of sides of Δabc and the fact that it is similar to Δade to solve for unknown sides of the smaller triangle. Since the area of the small square is equal to the sum of the areas of the trapezoid and a small triangle, subtract the area of Δade from the area of Δabc : 25 units^2 − 4.982 units^2 = 20.02 units^2.	Difficult

Synthesis Questions

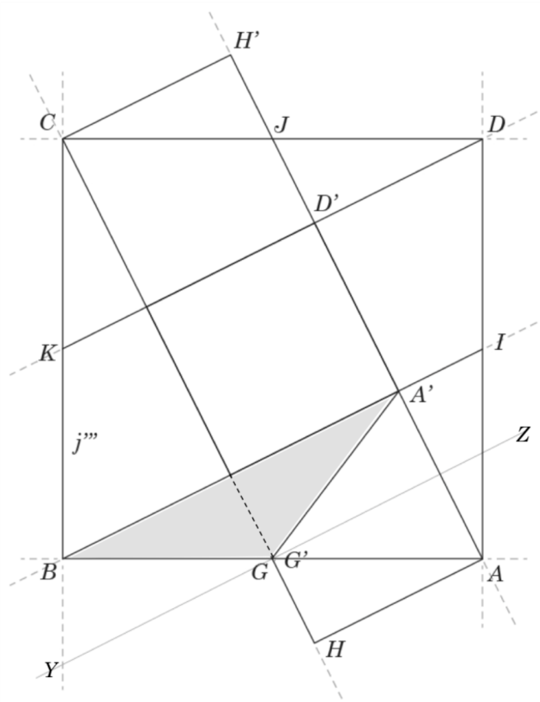
Below are sample responses to the questions in the Synthesis Questions section in the student handout.

1. How does the definition of congruent polygons allow you to prove congruence visually? What are some possible shortcomings of this kind of proof?

Polygons whose corresponding parts are congruent are congruent. To be congruent means to be the same or equal to. This allows two polygons to be superimposed, one on top of the other, so we can see that one exactly covers the other. This could only happen if all the parts of both shapes were equal. Using this visual method has the drawback that we must work very carefully to cut out two shapes perfectly, so they do “fit” each other. Any human imperfection in cutting or folding might make it difficult to see if the polygons were perfectly matched.

2. If a classmate unfamiliar with your work on this project saw the diagram below and stated that one of the angles formed by the segments appeared to be obtuse (for example ∠BGA’), how could you verify without using a protractor to measure that the apparent obtuse angle contains more than 90º?

There are isosceles (2 sides congruent), scalene (no sides congruent), right (contains 1 right angle), acute (all acute angles), and obtuse isosceles (contains one obtuse angle) triangles in the original square.



For the obtuse angle of the obtuse isosceles triangle ΔBGA’, one method is to construct a line parallel to the base of the obtuse triangle (segment BI ∥ YZ) that passes through the vertex of the obtuse angle, then alternate or vertical angles and their complements to show that the sum of the angles must be greater than 90 degrees (see diagram).

3. Integers that satisfy the Pythagorean equation for right triangles are called "Pythagorean triples." There are infinitely many such sets of integers, but the 3-4-5 right triangle is the smallest Pythagorean triple. The triangles you drew and used in first part of this project were multiples of the 3-4-5 right triangle. How can you confirm this?

We constructed triangles with perpendicular sides of 15 and 20 units, which gave a hypotenuse of 25 units, confirmed by the Pythagorean equation and also visually on graph paper. The 15-20-25 right triangle can be obtained by multiplying the lengths of the 3-4-5 right triangle by 5:

3 x 5 = 15

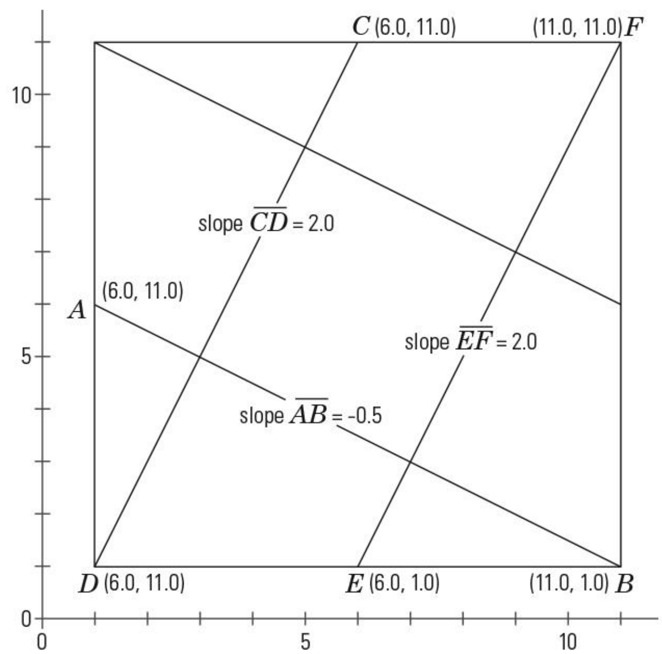
4 x 5 = 20

5 x 5 = 25

4. How can you prove algebraically that pairs of adjacent segments that form the smaller square are, in fact, perpendicular and that opposite pairs of segments are, in fact, parallel? Use additional graph paper as needed.

Two non-vertical lines are parallel if and only if they have the same slope. Two non-vertical lines are perpendicular if and only if they have slopes whose product is -1. This can be proven by plotting the lines that form the squares on a coordinate plane to determine their slopes by $m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$

(see diagram).



Extension and Real Life Application

Below are sample responses to the questions in the Extension and Real Life Application section in the student handout.

1. Discuss at least one real life application (and extension) of concepts relevant to this entire project.

Designers of quilts or tile work can use transformations to produce a tessellation that forms a pattern based on only a few design elements. For example, some kinds of Islamic art are based on the orderly, repeating patterns of polygons. They can be simple shapes or complicated ones, and can cover large plane surfaces such as walls, floors, and ceilings. Space can be filled with color or left blank so that the absence of color creates what artists call "negative space." Quilt makers rely on shapes as well as colors of fabric pieces to create an overall interesting design. In both cases, transformations include reflections of shapes, translations, rotations, or even dilations.

Zooming in and out of images, where the aspect ratio remains the same but size changes, is another application. For example, if an x-ray is taken of one's hand the doctor might need to zoom in on one small area to examine it for a break in a tiny bone. Zooming enlarges the bone in question without changing its aspect ratio. Digital imagery and software technology make this possible; in the past the doctor might have to take additional x-ray images with the lens closer to the body part, ion order to obtain a closer image. Digital technology allows for zooming to be done with less radiation administered to the patient.

2. Create a set of four congruent right triangles that satisfy the Pythagorean triple of 5-12-13, and use these triangles to construct a small, inner square as you did in investigation Question 4. What is the area, in square units, of the inner square?

The area of the inner square is 41.75 units^2.

3. Suppose a right triangle has its shortest side 8 units in length. If the dimensions of the perpendicular side and the hypotenuse complete a Pythagorean triple, what would be the area, in square units, of an inner square constructed as in the previous question?

The triangle would have sides of 8 and 15 units respectively, and hypotenuse of 17 units, according to the Pythagorean equation.

The area of the inner square is 38.1 units^2.

4. If you constructed the square from the right triangles in the previous question, what percent of the area of the largest possible square is the area of the smallest possible square you can make using the methods followed in this project?

The largest possible area is formed by the twisted inner square of a 23 x 23 square (the sum of the 8 + 15 units of the triangles' sides). The area of this inner twisted square is the square of the hypotenuse, $17^2 = 289$ units^2.

The smallest possible area is formed by the square made of overlapped triangles, from the previous question, 38.1 units^2. That is about 13% of the area of the largest twisted square.

5. The ancient Egyptians used ropes knotted at regular intervals to measure right angles. Three ropes were connected and each rope had a different number of knots tied at regular intervals. When pulled taut, a right triangle was formed which could be used to ensure that a stone block had each face perpendicular to the faces adjoining it. How many knots would have been tied in each rope? Explain your thinking.

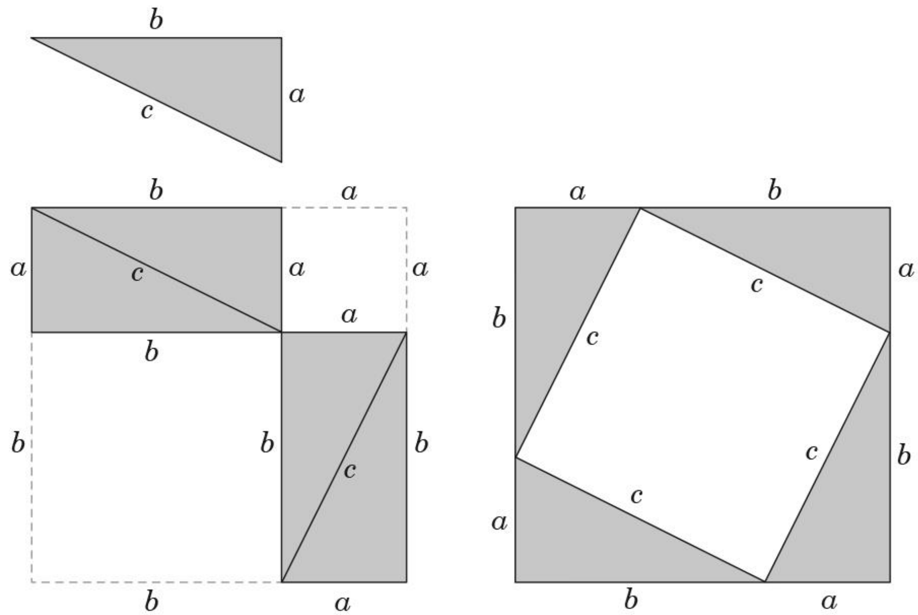
The Egyptians would have knotted ropes of 3 equally spaced intervals, 4 equally spaced intervals, and 5 equally spaced intervals. They would hold the ropes taut in a triangle, and align the 3-knot and 4-knot ropes with the stone block faces. This would verify the right angles at the corners.

6. Suppose you have eight congruent right, scalene triangles with sides of length *a*, *b*, and *c*. Show how you can use four of the triangles to create two congruent rectangles of length *a* and width *b*, and another

four of the triangles to create a square with sides length *c*. How can these figures be used to demonstrate the Pythagorean Theorem?

Place the two congruent rectangles perpendicular and touching at one vertex. Arrange two right triangles so one long side and one short side, respectively, are placed together to form one side of a larger square. Place a short side next to each long side until four sides of a larger square are formed, leaving a twisted, open square in the middle (see reference to "negative space" in previous question).

The larger square has an area that is the square of the sum of the two sides ($a + b$) of the triangle, while the smaller, twisted inner square has an area that is the square of the hypotenuse of the triangle. The sum of the areas of the square that is $a \times a$ and the square that is $b \times b$ is equal to the area of the larger square that is $c \times c$. Substituting values for *a, b,* and *c* (such as 3, 4, and 5 units respectively) yields a result that numerically agrees with the diagram shown.

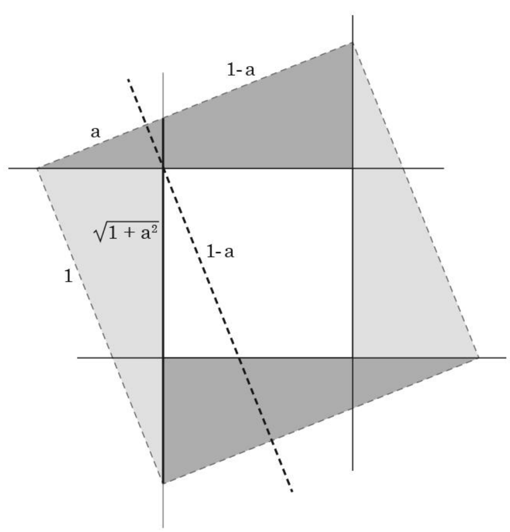


7. Suppose you can construct a point anywhere on the side of the larger unit square and connect that point to the opposite vertex. This point divides the side into segments of length *a* and (1 – *a*). Create the smaller, inner square in this manner by constructing congruent points and segments on each side of the larger square. What must be the ratio of the lengths *a* and (1 – *a*) such that the area of the smaller square is one-half that of the larger square?

Construct a segment through the vertex of the smaller square (the smaller square is shown here parallel to the edge of the page) that is parallel to the side of the larger square, as shown. The length of the segment connecting the vertex to the opposite side of the small square is $1 - a$. The length of the corresponding segment in the larger square is $\sqrt{1 + a^2}$.

The ratio of similarity then becomes $\dfrac{1 - a}{\sqrt{1 + a^2}}$.

The ratio of the areas of the two (similar) squares is the square of the similarity ratio:



$$\frac{(1-a)^2}{\left(\sqrt{1+a^2}\right)^2} = \frac{1-2a+a^2}{1+a^2}$$

Setting the ratio of the areas equal to ½ and solving for the required length of *a*:

$$\frac{1}{2} = \frac{1-2a+a^2}{1+a^2}$$

$$1+a^2 = 2\left(1-2a+a^2\right)$$

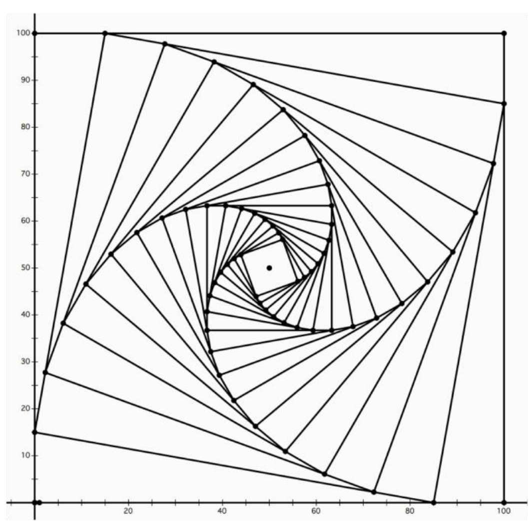
$$1+a^2 = 2-4a+2a^2$$

$$a^2-4a+1 = 0$$

$$a = 0.269$$

The other root of the quadratic equation, $a = 3.725$, can be disregarded because it exceeds the length of the side of the unit square.

8. An artist wants to create a sculpture based on multiple rotations of a square tile, stacked one on top of the other. She plans to make a series of squares that are successively smaller, and rotate them by some fixed amount as she stacks each new layer. She has seen a 2-dimensional design on a Cartesian coordinate system with the vertices of a square at points (0, 0), (100, 0), (100, 100), and (0, 100) respectively. She observes that a rotated inner square was formed by plotting the vertex of an inscribed square at points (0, *a*), which is some distance above the origin. Another vertex of this inscribed square has been plotted at (*a*, 100). This defines a right triangle whose short leg has length *a*, whose long leg has length *b*, and whose hypotenuse is one side of the inscribed square. What must be the ratio of *a* to *b* so that each new square can be rotated by a fixed amount when she stacks it on top of the one below it? What is the ratio of similarity for the squares?



The number of degrees of rotation is arbitrary. For a rotation of 10° clockwise, for example, the ratio of side *a* to side *b* is equal to the tangent of 10°.

$$\tan 10° = 0.1763 = \frac{a}{b}$$

$$a + b = 100$$

$$a = 100 - b$$

$$0.1763 = \frac{(100 - b)}{b}$$

$$0.1763b = 100 - b$$

$$0.1763b + b = 100$$

$$1.1763b = 100$$

$$b = 85.01$$

This means that the second set of vertices can be plotted at (0, 14.99), (14.99, 100), (100, 85.01), and (85.01, 100).

The sides of the second square are formed by the hypotenuses of the four triangular regions. Each triangle has sides *a* = 14.99 units and side *b* = 85.01 units. The length of each triangle's hypotenuse is 86.32 units:

$$a^2 + b^2 = c^2$$

$$14.99^2 + 85.01^2 = c^2$$

$$\sqrt{7451.4} = c$$

$$c = 86.32$$

Since the hypotenuse of the triangular portion forms the side of the rotated square, the scale factor may be determined by the ratio of the hypotenuse to the side of the original square:

$$\frac{\text{hypotenuse}}{\text{original side}} = \frac{86.32}{100}$$

$$0.8632$$

Therefore the artist should make each square 86.32% the size of the one before it, and stack the squares on top of each other rotated by 10° in order to build up the sculpture.

Mathematics Investigation Report – Assessment Rubric **Level I**		**Name:** **Math Class:**				
Component	To receive highest marks the student:	**4** **Expert**	**3** **Prac-titioner**	**2** **Appren-tice**	**1** **Novice**	**0** **No Attempt**
1. Preparation and Research	☐ Collaborated with group members on relevant research ☐ Brought and prepared all items necessary for their presentation ☐ Prepared supporting material for their presentation ☐ Prepared a thorough presentation for their classmates					
2. Demonstrations, Models, or Experiments	☐ Obtained all necessary materials and used them to explore the guiding questions of the investigation ☐ Completed and documented results, data, and observations for guiding questions of the investigation ☐ Performed experiments or demonstrations proficiently for others, or explained clearly a model and the concepts the student investigated with the model					
3. Content	☐ Presented written and spoken explanations that were mathematically accurate and paraphrased in the student's own words ☐ Answered all questions posed by their teacher or classmates correctly and thoughtfully ☐ Answered project synthesis and real-world application questions correctly and thoroughly ☐ Communicates clearly an understanding of the connection between their model or experiments and the driving question and theory behind the over-arching concept					
4. Real-World Application and Extension	☐ Identifies in written and spoken explanations the application of the topic to the real world, including specific examples ☐ Thoroughly discusses relevance of the topic to real life					
5. Collaboration and Contribution	☐ Group members equitably shared responsibility for: • Research of guiding questions • Presentation and discussion of results					
6. Presentation	☐ Delivered their content clearly and thoroughly, in an organized, logical manner with • Eye contact and poise • Appropriate voice level and clarity • Addressing the class/audience					
Total Points for Investigation (Maximum of 24 Points)						

<u>Guidelines for Marks:</u>

4 = **Expert:** Distinguished command of the topic; students show insightful and sophisticated communication of their understanding
3 = **Practitioner:** Strong command of the topic; students show reasonable and purposeful communication of their understanding
2 = **Apprentice:** Moderate command of the topic; students show adequate but basic communication of their understanding
1 = **Novice:** Partial command of the topic; students show limited and insufficient communication of their understanding
0 = **No Attempt**